

Networks and Graphs: Spanning Trees**VII.B Student Activity Sheet 8: Kruskal's Algorithm**

A graph whose edges are given numerical values is called a *weighted graph*. Keeping all the vertices connected by a path resulting in a minimum total weight is called *finding a minimal spanning tree*. The word *spanning* means that each vertex remains connected to the graph, and the word *tree* indicates that there are no cycles.

The following procedure, known as Kruskal's Algorithm, can be used to find a minimal spanning tree in a weighted graph.

Kruskal's Algorithm

Assume that you start with a table of the weights associated with each edge (just like the Railroad problem in Student Activity Sheet 7).

Step 1: Put all of the weights in a list from smallest to largest.

Step 2: Find the smallest weight in the list and include the associated edge and two vertices, as long as that does not create a cycle.

Step 3: Remove this weight from the list.

Step 4: Repeat Steps 2 and 3 until all vertices are connected.

1. A series of bridges will be constructed to connect a group of seven islands. The highway department wants to make sure that a vehicle can be driven from one island to the others in this new network of bridges. The cost of building a bridge is directly proportional to the length of the bridge. The following table provides the distances in miles between each pair of islands.

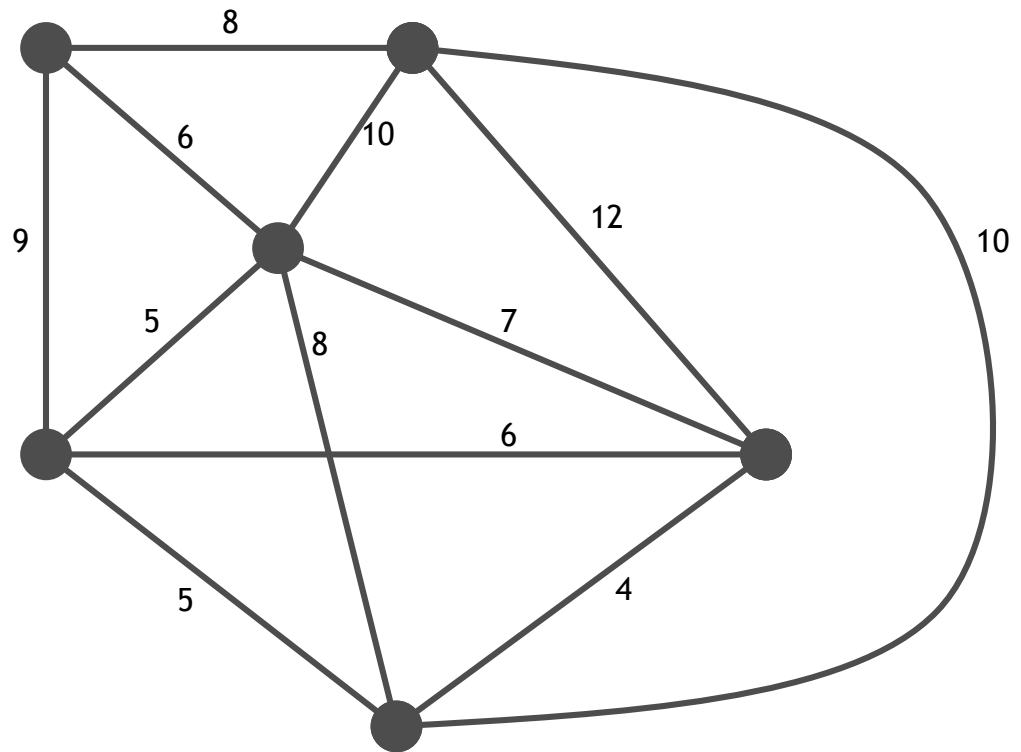
	Island A	Island B	Island C	Island D	Island E	Island F	Island G
Island A		10	8	8	7	10	9
Island B			4	9	13	3	7
Island C				12	11	5	9
Island D					9	10	6
Island E						6	11
Island F							8

Use Kruskal's Algorithm to determine which islands should be connected by bridges. Draw a graph that represents the seven islands with the bridges that will be constructed.

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2. Use Kruskal's Algorithm to find a minimal spanning tree in the following graph.



3. **REFLECTION:** Do all graphs have spanning trees? Are spanning trees unique?

4. **EXTENSION:** Other algorithms exist for arriving at a minimal spanning tree. Conduct research to find one and share it with your class through a brief presentation. Compare and contrast your findings with Kruskal's Algorithm as appropriate.